Inferring Connectivity for Computational Neural Network Models

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INTRODUCTION

Modelling the causal relationship within a dynamic system is an ill-posed mathematical problem and impractical task to implement through deterministic methods. The need to employ tools of probability are vital to create stochastic models that can accurately predict to a degree of certainty what the dynamics of a system is telling us. In order to infer the over behavior of a system, it is also important to deconstruct the way in which it is connected. A useful parallel that many of the modelling techniques are derived from is the structure of two biological neurons. Where one is a presynaptic neuron directed towards a post-synaptic neuron.

BACKGROUND

The notion of causality was original proposed by Weiner but was not formalized until Granger [2] published a more rigorous definition represented with a vector auto regressive(VAR) model.

Granger's formulation for predicting $\boldsymbol{X_t}$ given only the history of a time-series $\boldsymbol{X}$ is given by

$$\boldsymbol{X}_t = \sum_{k=1}^{n} \boldsymbol{A}_k \boldsymbol{X}_{t-k} + \boldsymbol{\epsilon}_t \qquad (1)$$

Where $\boldsymbol{X_t}$ is the value of the time series at time t and $\boldsymbol{A_k}$ is the regression coefficient vector, and $\boldsymbol{\epsilon_t}$ is the error, or residual, for our prediction of time $\boldsymbol{X_t}$.

However, to determine if there is causal relationship between a time series $\boldsymbol{X}$ and another time series $\boldsymbol{Y}$, we must compare the performance of the equation

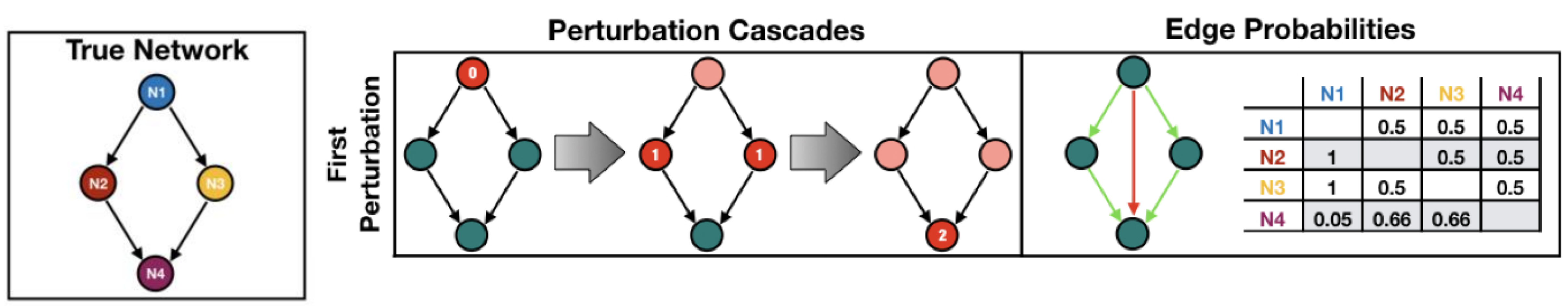
$$\boldsymbol{X}_t = \sum_{k=1}^{p} \boldsymbol{A}'_k \boldsymbol{X}_{t-k} + \sum_{k=1}^{p} \boldsymbol{B}' \boldsymbol{Y}_{t-k} + \boldsymbol{\epsilon}'_t \qquad (2)$$

Similar to eq. (**1**), to predict a value at some time t, we use the VAR model that is inclusive for both time series data $\boldsymbol{X}$ and $\boldsymbol{Y}$.

Generally, if it is possible to calculate the a more accurate prediction of $\boldsymbol{X}$ using $\boldsymbol{Y}$ and the appropriate statistical test indicates significance, then we say $\boldsymbol{Y} \rightarrow \boldsymbol{X}$. Read as $\boldsymbol{Y}$ causes $\boldsymbol{X}$.

CAUSAL ALGORITHM

Figure 1: Perturbation Cascade Inference



METHOD

Dimensionless Leaky-integrate-and-fire model

The model for the biologically plausible neural dynamics to be used will be a dimensionless version of the leaky-integrate-and-fire (LIF) model. This conductance-based model is identical to the model for a resistor-capacitor (RC) circuit and is widely used to its simplicity for numerical integration. The simple LIF model is as follows

$$c_m \frac{dV_j}{dt} = -g_m(V_j - V_{reset}) + I_{app} \qquad (3)$$

Where c_m is the membrane capacitance of the cell, g_m is the membrane conductance, V_j is the transmembrane potential of the j^{th} cell, and with V_{reset} being the reset threshold for the firing of a neuron.

The specific variant of this model is the dimensionless LIF proposed by Lewis and Rinzel [3]. This allows the transformation of (**3**) into

$$\frac{dV_i}{dt} = -V_i + \boldsymbol{I}_{app} + \sum_k A_{ki} \boldsymbol{I}_{syn} + \sum_{k \in C_i} g_c((V_k - V_i) + \beta \sum_j \delta(t - \tau_j)) \qquad (4)$$

And the synapse as

$$I_{syn} = \sum_j \alpha^2 \exp[\alpha(t - \tau_j)]\,(t - \tau_j) B_{ji} \qquad (5)$$

Kuramoto model

Using the nonlinear harmonic Kuramoto [4] model, we can exhibit dynamics approximately like that of biological neurons and generalized oscillators, using

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{n} \sum_{j=1}^{k} \mathrm{A_{ij}} \sin(\theta_j - \theta_i) + f(t), \quad 1 \le i \le n. \qquad (6)$$

Where ω_i is our natural frequency, K is the coupling strength, θ represents the phase, and A_{ij} is the $n \times n$ connectivity matrix for our network size n with a forcing term $f(t)$.

Calculating the Accuracy for Network Performance

We can present the performance with a confusion matrix given by

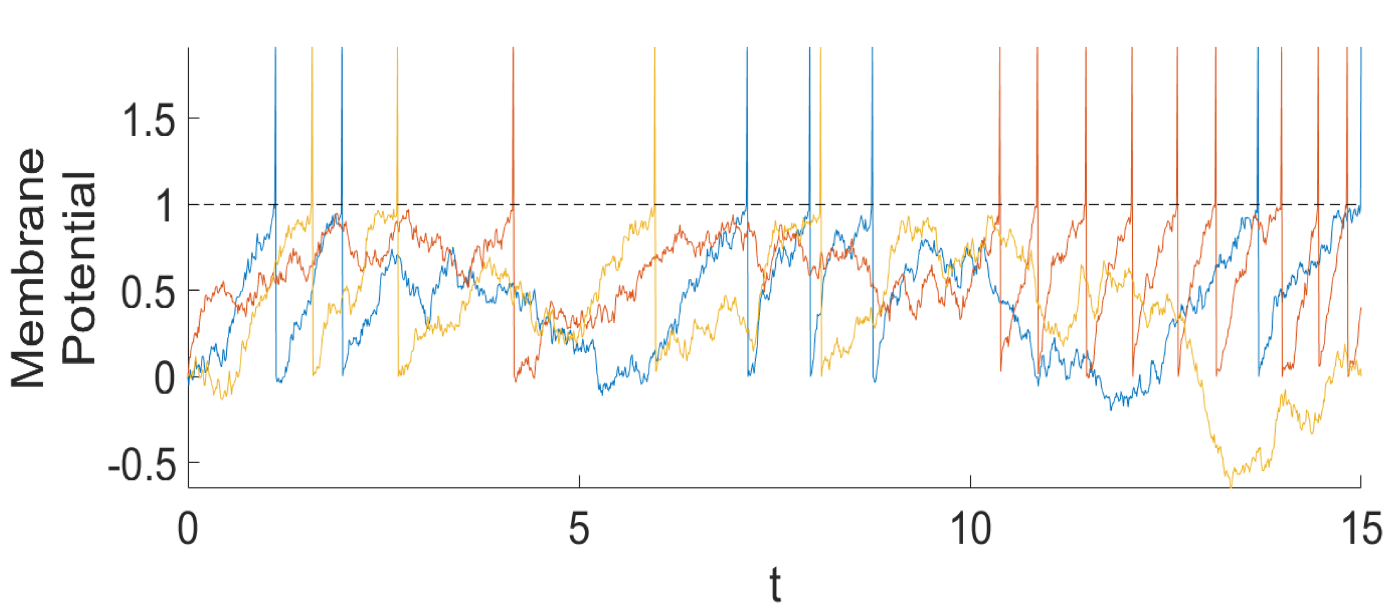
$$\begin{bmatrix} TP & FN \\ FP & TN \end{bmatrix}$$

Where TP and TN represent the count of true inferred present and absent connections. Conversely, FP and FN represent the count of falsely inferred presence and absence of connections.

We can calculate the accuracy of our model by taking the sum of all true inferences over the sum of all inferences made, given by

$$Accuracy = Acc(\phi) = \frac{TP + TN}{TP + TN + FP + FN} \qquad (7)$$

Figure 2: graph of realistic dynamics modeled through the Dimensionless LIF



RESULTS

Figure 3: Performance for a perturbation strength of 10 N

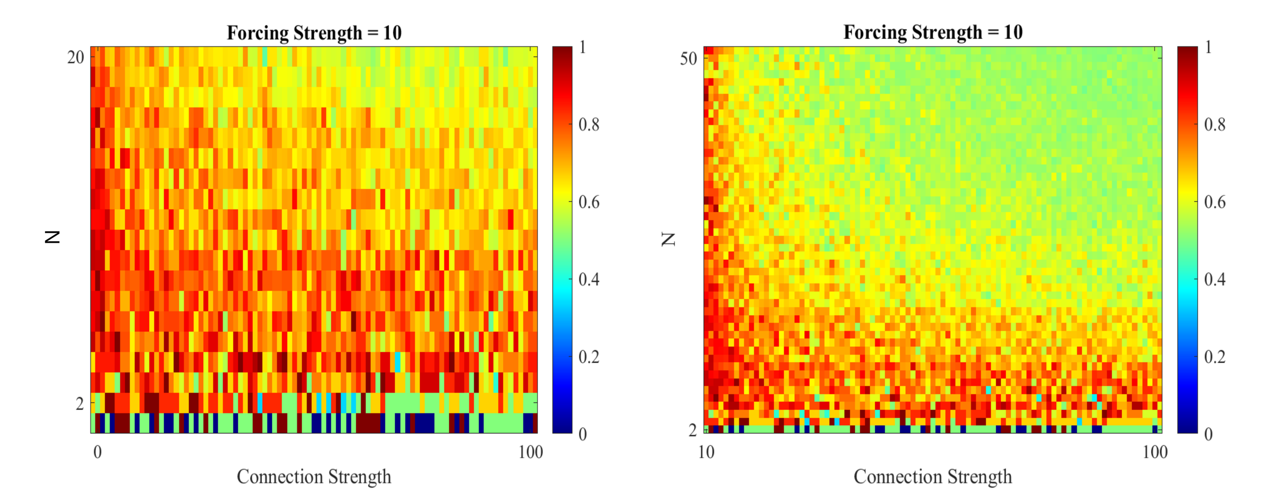
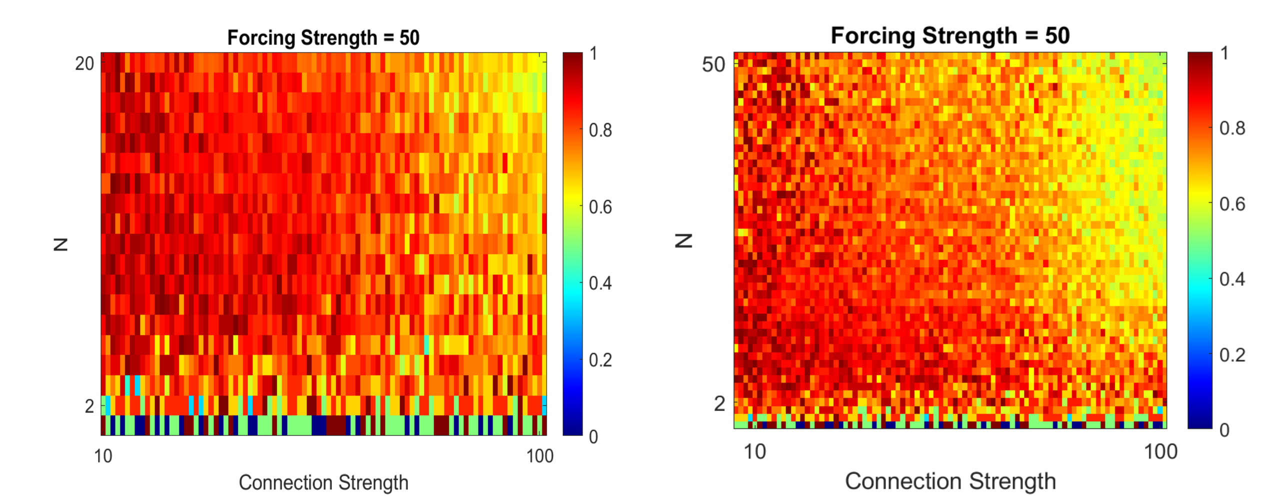


Figure 4: Performance for a perturbation strength of 50 N



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